

- Consider the following general population regression function (PRF):

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t \quad \text{for } t = 1, 2, \dots$$

where the classical assumption of no autocorrelation (or no serial correlation) indicates that:

$$E(u_t \cdot u_{t-j}) = 0 \quad \text{with } j \geq 1.$$

- The violation of this assumption implies that there is a relationship between u_t and u_{t-j} as follows:

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \dots + \rho_p u_{t-p} + \varepsilon_t \quad (1)$$

- ρ_1 is the first-order autocorrelation coefficient,
- ρ_2 is the second-order autocorrelation coefficient,
- ρ_1 is the first-order autocorrelation coefficient,
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- ρ_p is the p^{th} -order autocorrelation coefficient.

- Now we want to test the null hypothesis of no autocorrelation problem; that is,

$$\left\{ \begin{array}{ll} H_0: \rho_1 = \rho_2 = \rho_3 = \dots = \rho_p = 0 & \text{No autocorrelation problem} \\ H_1: \text{At least one of these } \rho\text{'s} \neq 0. & \text{Otherwise} \end{array} \right.$$

- Here we use the LM test. This test is derived from the Lagrange Multiplier (LM) principle.

The LM test equation (for $Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$) is:

$$e_t = a_1 + a_2 X_{2t} + a_3 X_{3t} + \rho_1 e_{t-1} + \rho_2 e_{t-2} + \rho_3 e_{t-3} + \dots + \rho_p e_{t-p} + \varepsilon_t$$

- Note that u_t (in $u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \dots + \rho_p u_{t-p} + \varepsilon_t$) is not observable, and thus it is replaced by the residuals e_t .
- In addition, we control for the independent variables by including them in the LM test equation.

Failure to reject $H_0: \rho_1 = \rho_2 = \rho_3 = \dots = \rho_p = 0$ indicates that, overall, there exists no autocorrelation problem.

The LM test equation is: $e_t = a_1 + a_2 X_{2t} + a_3 X_{3t} + \rho_1 e_{t-1} + \rho_2 e_{t-2} + \rho_3 e_{t-3} + \dots + \rho_p e_{t-p} + \varepsilon_t$

$H_0: \rho_1 = \rho_2 = \rho_3 = \dots = \rho_p = 0$ No autocorrelation problem

H_1 : At least one of these ρ 's $\neq 0$. Otherwise

The decision rule is:

Reject $H_0: \rho_1 = \rho_2 = \rho_3 = \dots = \rho_p = 0$ if

the p -value of the calculated F -value (from the LM test output) ≤ 0.10 .

- We further need to do the following individual tests using the t-test:

$$\left\{ \begin{array}{l} H_0: \rho_1 = 0 \\ H_1: \rho_1 \neq 0 \end{array} \right\} \quad \left\{ \begin{array}{l} H_0: \rho_2 = 0 \\ H_1: \rho_2 \neq 0 \end{array} \right\} \quad \left\{ \begin{array}{l} H_0: \rho_3 = 0 \\ H_1: \rho_3 \neq 0 \end{array} \right\} \quad \dots \quad \left\{ \begin{array}{l} H_0: \rho_p = 0 \\ H_1: \rho_p \neq 0 \end{array} \right\}$$

- Failure to reject $H_0: \rho_1 = 0$ indicates that there exists no 1st-order autocorrelation problem; failure to reject $H_0: \rho_2 = 0$ indicates that there exists no 2nd-order autocorrelation problem, and so on.